

# Mr. Wright's Math Extravaganza

## **Precalculus**

## **Three Dimensional Analytic Geometry**

Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: level 3.0 and success on applications

Score	I Can Statements		
4.0	□ I can demonstrate in-depth inferences and applications that go beyond what was taught.		
3.5	In addition to score 3.0 performance, partial success at score 4.0 content		
	□ I can graph points and equations in a three-dimensional rectangular coordinate system.		
3.0	I can use vector operations in three dimensions.		
	I can write equations for lines and planes in three dimensions.		
2.5	No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content		
2.0	I can calculate 3D distance and midpoint.		
	I can graph spheres.		
	I can find the angle between vectors.		
	I can evaluate a cross product.		
	I can find the angle between two planes.		
	I can graph planes.		
1.5	Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content.		
1.0	With help, partial success at score 2.0 content and score 3.0 content.		
0.5	With help, partial success at score 2.0 content but not at score 3.0 content.		
0.0	Even with help, no success.		

# Precalculus

### 11-01 3-D Coordinate System



# Precalculus

## 11-02 Vectors in space

#### Vectors in 3-D

	$\vec{v} = \langle v_1, v_2, v_2 \rangle$		
• To find a vector from the	$v = (v_1, v_2, v_3)$ noint $(n_1, n_2, n_3)$ to the	point $(a_1, a_2, a_3)$	
	$\vec{v} = \langle a_1 - n_1, a_2 - n_2, a_3 - n_2 \rangle$	point (q1, q2, q3)	
If $\vec{v} = \langle v_1, v_2, v_2 \rangle$ and $\vec{u} = \langle u_1, u_2, u_2 \rangle$ .	· (41 P1)42 P2)43 P3)		
Addition			
• Add corresponding			
	$\vec{v} + \vec{u} = \langle v_1 + u_1, v_2 + u_2, v_3 + u_3 \rangle$		
• Scalar multiplication	× 1 1' 2 2' 3 3'		
o			
	$c\vec{v} = \langle cv_1, cv_2, cv_3 \rangle$		
Dot Product	1 2 3		
	$\vec{v} \cdot \vec{u} = v_1 u_1 + v_2 u_2 + v_3 u_3$		
Magnitude			
	$\ \vec{v}\  = \sqrt{v_1^2 + v_2^2 + v_3^2}$		
• Unit vector in the direction of $\vec{u}$	N		
• Onit vector in the direction of <i>v</i>	11		
• Angle between vectors			
· migie between vectors	$\vec{u} \cdot \vec{v} = \ \vec{u}\  \ \vec{v}\  \cos \theta$		
• If $\theta = 90^\circ$ (and $\vec{u} \cdot \vec{v} =$	) then vectors are		
• If $\vec{u} = c\vec{v}$ , then vectors are	), alon vectore al c		
Let $\vec{m} = \langle 1 \ 0 \ 3 \rangle$ and $\vec{n} = \langle -2 \ 1 \ -4 \rangle$			
Find $\ \vec{m}\ $	Find unit vector in direct	tion of $\overline{m}$	
Find $\vec{m} + 2\vec{n}$	Find $\vec{m} \cdot \vec{n}$		
Find the angle between $\overline{m}$ and $\overline{n}$			

Are  $\vec{p} = \langle 1, 5, -2 \rangle$  and  $\vec{q} = \left\langle -\frac{1}{5}, -1, \frac{2}{5} \right\rangle$  parallel, orthogonal, or neither?

Are P(1, -1, 3), Q(0, 4, -2), and R(6, 13, -5) collinear?

# Precalculus

## 11-03 Cross Products

#### **Cross Product**

- $\hat{\iota}$  is \_\_\_\_\_\_vector in x,  $\hat{j}$  is unit vector in y, and  $\hat{k}$  is unit vector in z
- $\vec{u} = u_1 \hat{\iota} + u_2 \hat{j} + u_3 \hat{k}$  and  $\vec{v} = v_1 \hat{\iota} + v_2 \hat{j} + v_3 \hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{\iota} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

If  $\vec{u} = \langle -2, 3, -3 \rangle$  and  $\vec{v} = \langle 1, -2, 1 \rangle$ , find  $\vec{u} \times \vec{v}$ 

#### **Properties of Cross Products**

- $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$
- $\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w})$
- $c(\vec{u} \times \vec{v}) = c\vec{u} \times \vec{v} = \vec{u} \times c\vec{v}$
- $\vec{u} \times \vec{u} = 0$
- If  $\vec{u} \times \vec{v} = 0$ , then  $\vec{u}$  and  $\vec{v}$  are parallel
- $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$
- $\vec{u} \times \vec{v}$  is orthogonal to  $\vec{u}$  and  $\vec{v}$
- $\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \|\vec{v}\| \sin \theta$

#### Area of a Parallelogram

 $\|\vec{u} \times \vec{v}\|$  where  $\vec{u}$  and  $\vec{v}$  represent adjacent sides



$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

#### Volume of Parallelepiped

 $V = |\vec{u} \cdot (\vec{v} \times \vec{w})|$  where  $\vec{u}$ ,  $\vec{v}$ , and  $\vec{w}$  represent adjacent edges



PO

 $\vec{n} = \langle a, b, c \rangle$ 

ax + by + cz + d = 0

 $P(x_1, y_1, z_1)$ 

Q(x, y, z)

 $\langle a, b, c \rangle$ 

## Precalculus

## 11-04 Lines and Planes in Space

Lines

## General form

$$\langle x - x_1, y - y_1, z - z_1 \rangle = \langle at, bt, ct \rangle$$

#### Parametric Equations of Line

 $x = at + x_1$   $y = bt + y_1$  $z = ct + z_1$ 

### Symmetric Equation of Line

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Find a set of parametric equations of the line that passes through (1, 3, -2) and (4, 0, 1).

Planes

Standard form

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

**General form** 

$$ax + by + cz + d = 0$$

Find the general equation of plane passing through A(3, 2, 2), B(1, 5, 0), and C(1, -3, 1)

