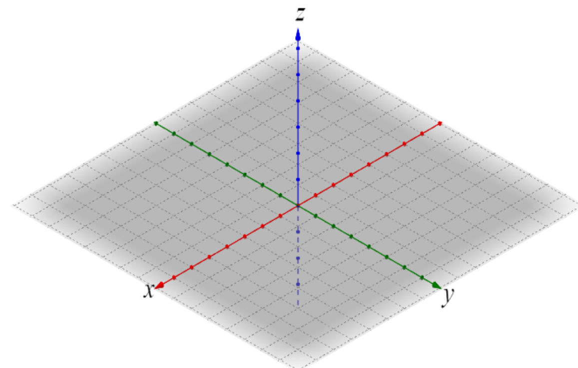
Precalculus

11-01 3-D Coordinate System

Points in 3 dimensions

- (x, y, z)
- Graph by moving out the __________, over the __________, then up the __________.

Graph A(5, 6, 3) and B(−2, −4, 0)



Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

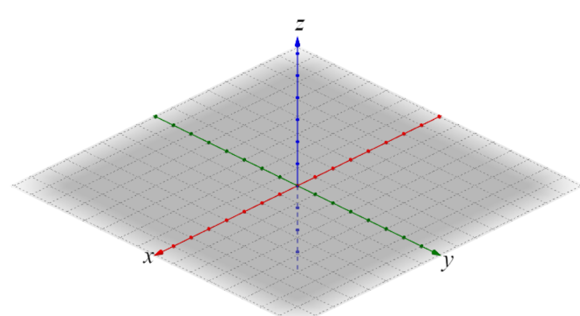
Equation of Sphere

$$(x - h)^2 + (y - k)^2 + (z - j)^2 = r^2$$

Center is (h, k, j), r = radius

- Graph by plotting the __________ and moving each direction the __________

Graph $(x - 2)^2 + (y + 1)^2 + (z + 1)^2 = 16$



Precalculus

11-02 Vectors in space

Vectors in 3-D

$$\vec{v} = \langle v_1, v_2, v_3 \rangle$$

- To find a vector from the _______________point (p_1, p_2, p_3) to the _______________point (q_1, q_2, q_3)

$$\vec{v} = \langle q_1 - p_1, q_2 - p_2, q_3 - p_3 \rangle$$

If $\vec{v} = \langle v_1, v_2, v_3 \rangle$ and $\vec{u} = \langle u_1, u_2, u_3 \rangle$,

- Addition
  - Add corresponding _______________

$$\vec{v} + \vec{u} = \langle v_1 + u_1, v_2 + u_2, v_3 + u_3 \rangle$$

- Scalar multiplication
  - _______________

$$c\vec{v} = \langle cv_1, cv_2, cv_3 \rangle$$

- Dot Product

$$\vec{v} \cdot \vec{u} = v_1u_1 + v_2u_2 + v_3u_3$$

- Magnitude

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

- Unit vector in the direction of $\vec{v}$

$$\frac{\vec{v}}{\|\vec{v}\|}$$

- Angle between vectors

$$\vec{u} \cdot \vec{v} = \|\vec{u}\|\|\vec{v}\| \cos\theta$$

- If $\theta = 90°$ (and $\vec{u} \cdot \vec{v} =$_______________), then vectors are _______________
- If $\vec{u} = c\vec{v}$, then vectors are _______________

Let $\vec{m} = \langle 1,\ 0,\ 3 \rangle$ and $\vec{n} = \langle -2,\ 1,\ -4 \rangle$

Find $\|\vec{m}\|$

Find unit vector in direction of $\vec{m}$

Find $\vec{m} + 2\vec{n}$

Find $\vec{m} \cdot \vec{n}$

Find the angle between $\vec{m}$ and $\vec{n}$

Are $\vec{p} = \langle 1,\ 5,\ -2 \rangle$ and $\vec{q} = \left\langle -\frac{1}{5},\ -1, \frac{2}{5} \right\rangle$ parallel, orthogonal, or neither?

Are $P(1,\ -1,\ 3)$, $Q(0,\ 4,\ -2)$, and $R(6,\ 13,\ -5)$ collinear?

Precalculus

11-03 Cross Products

Cross Product

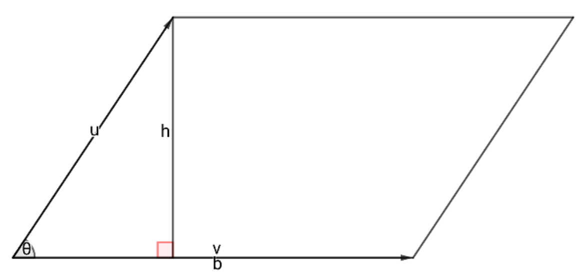
- $\hat{\imath}$ is _______________vector in x, $\hat{\jmath}$ is unit vector in y, and $\hat{k}$ is unit vector in z
- $\vec{u} = u_1\hat{\imath} + u_2\hat{\jmath} + u_3\hat{k}$ and $\vec{v} = v_1\hat{\imath} + v_2\hat{\jmath} + v_3\hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

If $\vec{u} = \langle -2,\ 3,\ -3\rangle$ and $\vec{v} = \langle 1,\ -2,\ 1\rangle$, find $\vec{u} \times \vec{v}$

Properties of Cross Products

- $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$
- $\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w})$
- $c(\vec{u} \times \vec{v}) = c\vec{u} \times \vec{v} = \vec{u} \times c\vec{v}$
- $\vec{u} \times \vec{u} = 0$
- If $\vec{u} \times \vec{v} = 0$, then $\vec{u}$ and $\vec{v}$ are parallel
- $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w}$
- $\vec{u} \times \vec{v}$ is orthogonal to $\vec{u}$ and $\vec{v}$
- $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\| \sin\theta$



Area of a Parallelogram

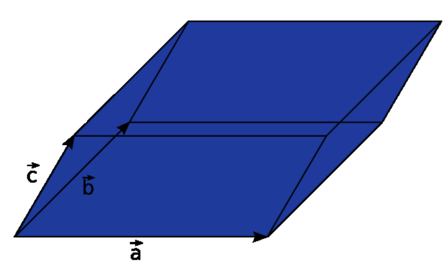
$\|\vec{u} \times \vec{v}\|$ where $\vec{u}$ and $\vec{v}$ represent adjacent sides

Triple Scalar Product (shortcut)

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Volume of Parallelepiped

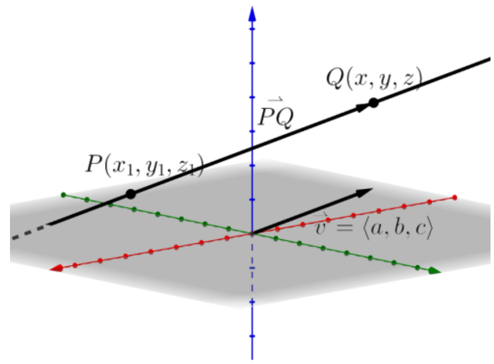
$V = |\vec{u} \cdot (\vec{v} \times \vec{w})|$ where $\vec{u}$, $\vec{v}$, and $\vec{w}$ represent adjacent edges



Precalculus

11-04 Lines and Planes in Space

Lines



General form

$$\langle x - x_1,\ y - y_1,\ z - z_1\rangle = \langle at,\ bt,\ ct\rangle$$

Parametric Equations of Line

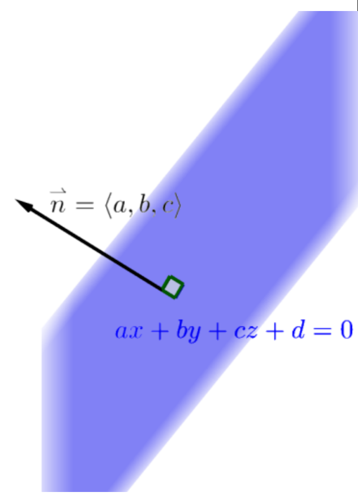
$$x = at + x_1$$
$$y = bt + y_1$$
$$z = ct + z_1$$

Symmetric Equation of Line

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Find a set of parametric equations of the line that passes through $(1, 3, -2)$ and $(4, 0, 1)$.

Planes



Standard form

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

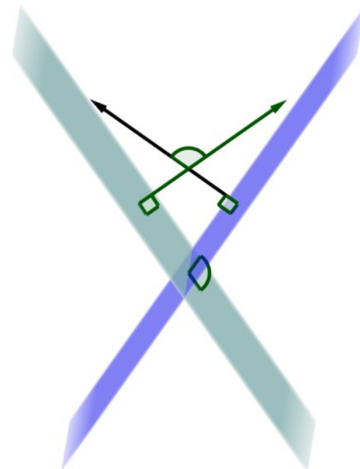
General form

$$ax + by + cz + d = 0$$

Find the general equation of plane passing through $A(3,\ 2,\ 2)$, $B(1,\ 5,\ 0)$, and $C(1,\ -3,\ 1)$

Precalculus 11-04 Name: ________________

Angle between two planes



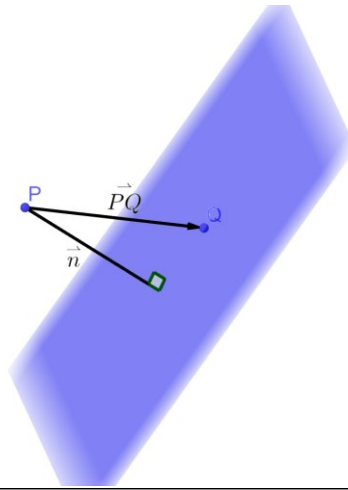
- Find the angle between _______________vectors
- Normal vectors are _______________in the equations of the plane

$$|\overrightarrow{n_1} \cdot \overrightarrow{n_2}| = \|\overrightarrow{n_1}\|\|\overrightarrow{n_2}\| \cos\theta$$

Distance between a Point and a Plane

$$D = \|proj_{\vec{n}}\, \overrightarrow{PQ}\|$$

$$D = \frac{|\overrightarrow{PQ} \cdot \vec{n}|}{\|\vec{n}\|}$$



Graphing planes in space

1. Find the _______________
2. _______________ the intercepts
3. Draw a _______________to represent the plane

Sketch $3x + 4y + 6z = 24$